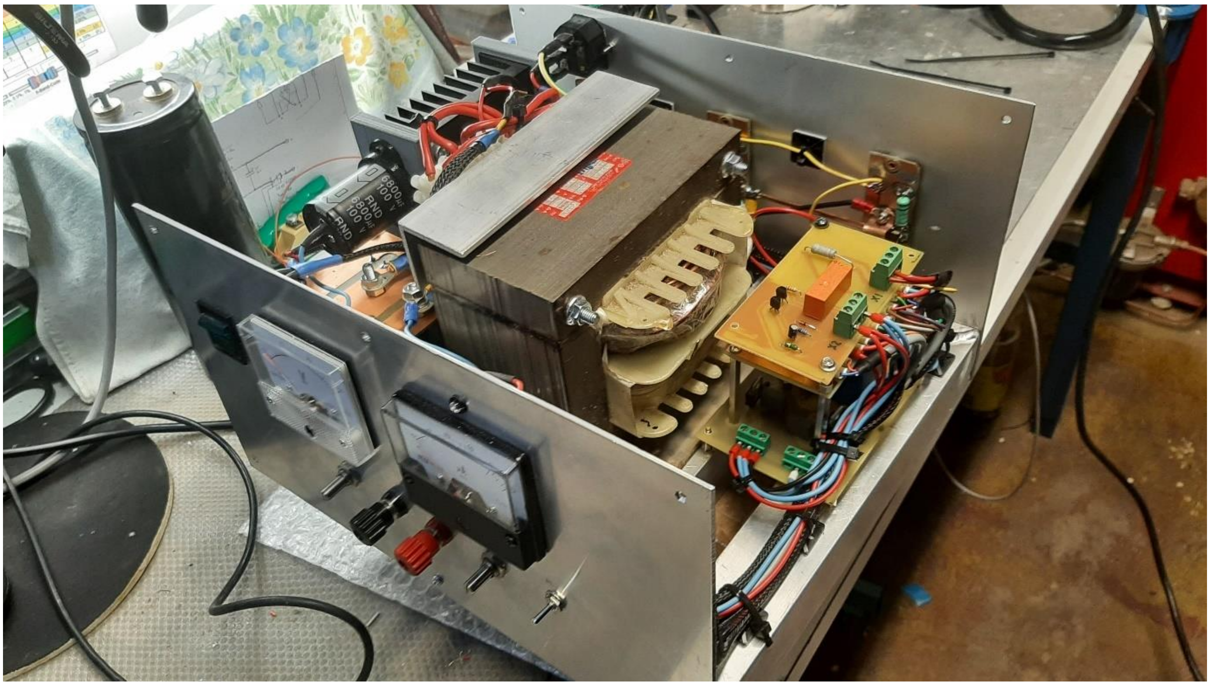
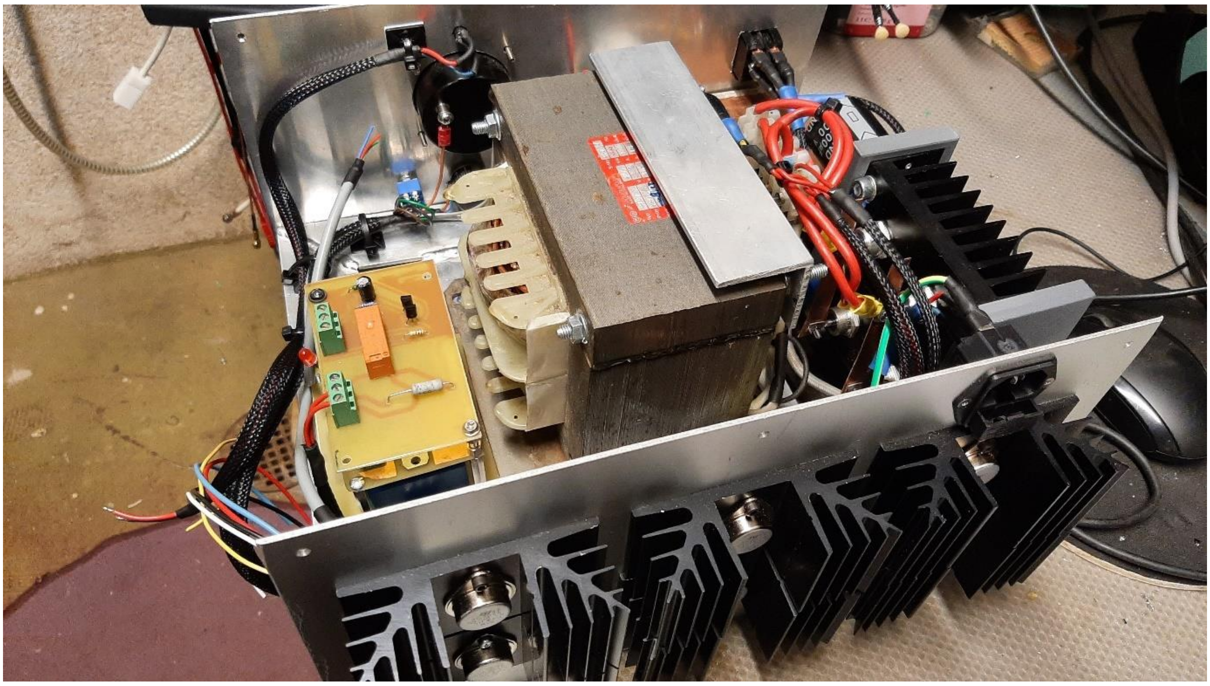




LINEAR POWER SUPPLY
30A

Table of contents

Design and build of a linear power supply

Target: Design and build a linear (transformer-based) power supply with adjustable regulated output and current limit with following parameters:

- ✓ Input voltage: 230V 50 Hz
- ✓ Output voltage: Adjustable, range about 5-22V)
- ✓ Output current: 30A DC continuous
- ✓ Voltage and current monitoring
- ✓ Current limit function

1. Selection of topology

Based on an available power transformer that could be modified to the intended purpose the following topology for the power supply was selected.

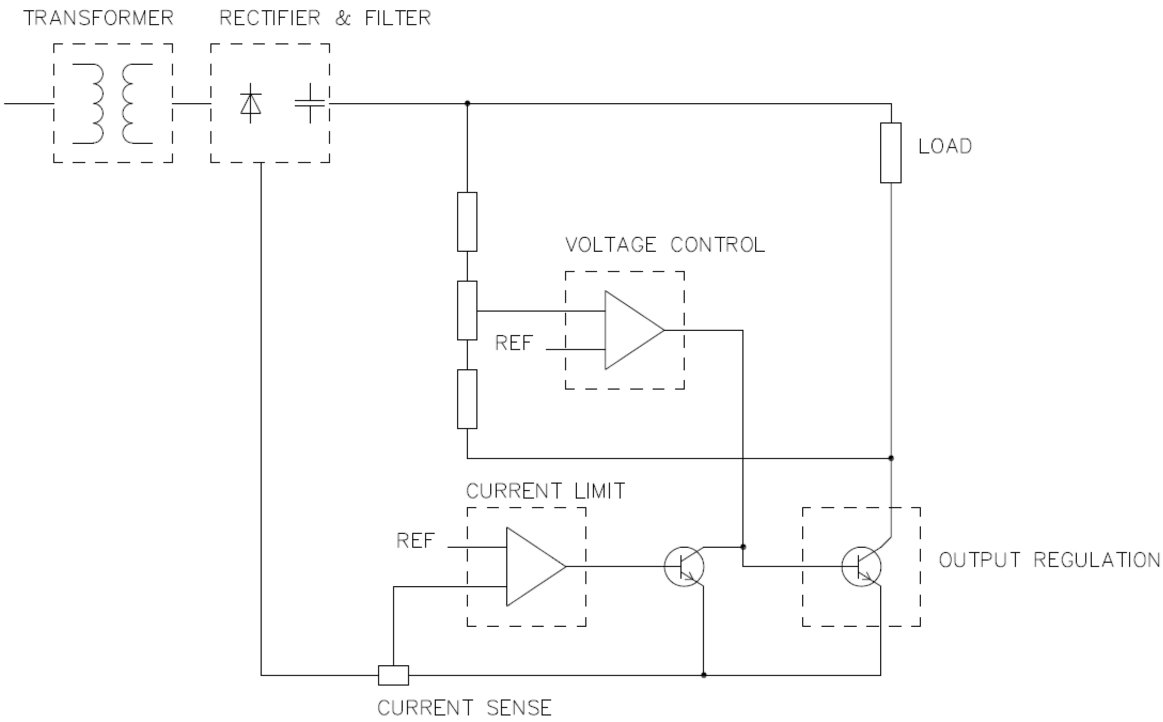


Figure 1:Topology

A conventional full-wave rectifier with sufficient filtering will provide the unregulated DC supply for the regulator. Regulation at the negative supply side of the load will be implemented by controlling output NPN-transistors' base current based on the voltage setpoint. In the DC return line, a current sense unit (LEM LA-50P, 1:1000) will provide a current measurement which will be compared with current limit setpoint. Current limit comparator will shut off the output transistor base drive thus providing a current limiting function. BJT transistors 2N3055 were selected for the output regulation.

2. General design and selection of main components

2.1 Transformer design

Standard transformers can be used, or special ones ordered according to requirements. Sometimes an existing transformer can be modified for hobbyist purposes. A procedure to design or modify a power transformer is presented here.
“Transformer and inductor design handbook” by Colonel Wm. T. McLyman, ISBN: 0-8247-5393-3 has been used as a reference.

2.1.1 Fixing transformer initial parameters

Parameter	Symbol	Value	Unit	Notes
Input voltage	V_{in}	230	V	
Output voltage	V_o	18	V	See below note 1
Output current	I_o	25	A	
Current density	J	2,5	A/mm^2	
Output power	P_o	450	W	
Frequency	f	50	Hz	
Efficiency	η	90	%	
Regulation	α	5	%	
Operating magnetic flux density peak value	B_{ac}	1,1	T	Typical for silicon steel at low frequencies
Window Utilization factor	K_u	0,4		
Temperature rise goal	T_r	30	°C	

Table 1:Design parameters

Note 1:
Requirement for the transformer secondary voltage: At full load, the filtered unregulated voltage shall be sufficiently high and not drop under the maximum output voltage setpoint at the end of the discharge cycle. Voltage drop due to filter capacitor dimensioning and two rectifier diode forward voltage drops must be accounted for. On the other hand, excess supply voltage will be dropped across the output regulation transistor collector-emitter and dissipated as power loss. Therefore, this sets the requirement for transformer secondary voltage maximum.

$$\widehat{V_o}(min) = V_{DCmax} + 2 * V_D + \Delta V_s = 16V + 2 * 1{,}2V + 4V = 22{,}4V \rightarrow 25V$$

Diode voltage drops for 2*1,2V and ripple of 4V at full load are presumed here. Therefore, as the supply voltage is sinusoidal, the secondary voltage RMS value shall be

$$V_0 = \frac{\widehat{V_o}(min)}{\sqrt{2}} = \frac{25V}{\sqrt{2}} = 17{,}7V \approx 18V$$

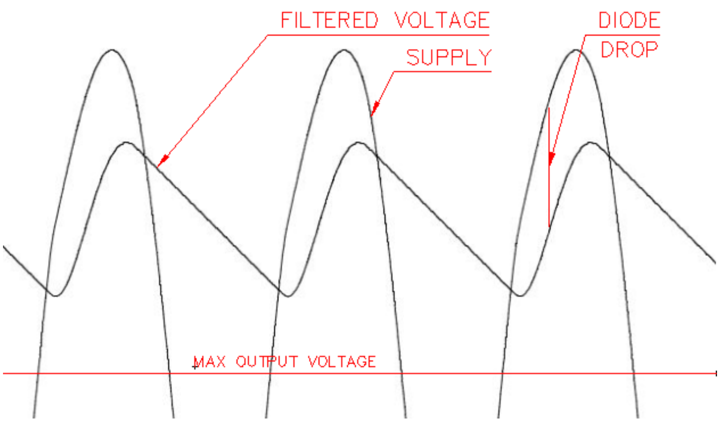


Figure 2:Secondary voltage selection

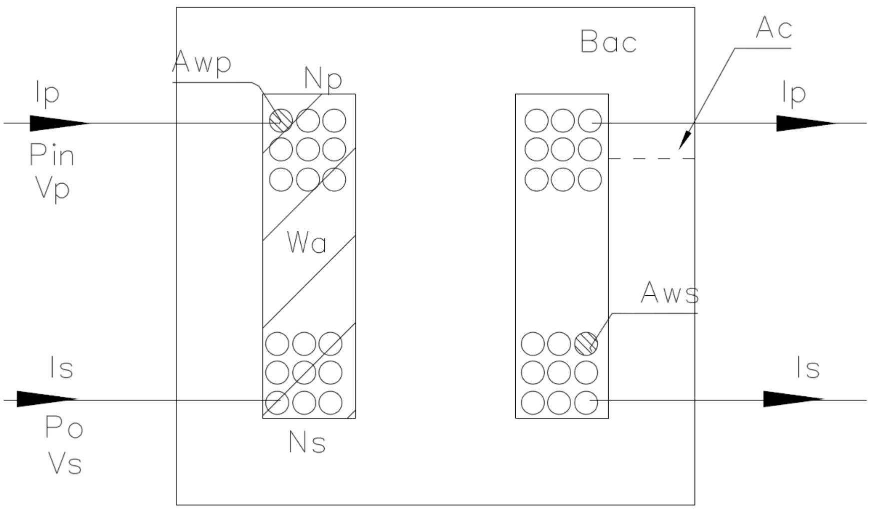


Figure 3:Transformer core (EI-core)

2.1.2 Deriving the Area Product

According to Faraday's law

$$V_p = K_f f N_p \hat{B}_{ac} A_c \rightarrow N_p = \frac{V_p}{A_c \hat{B}_{ac} f K_f}$$

where

V_p= Primary voltage (RMS)
N_p = Number of turns in the primary winding
A_c= Iron core cross-sectional area
K_f= waveform factor, for sinusoidal voltage $K_f = \frac{2\pi}{\sqrt{2}}$ (see 2.1.4)
$\hat{B}_{ac}$= Flux density peak value
f= Frequency
also

$$K_u W_a = N_p A_{wp} + N_s A_{ws}$$

where

K_u= Window utilization factor, total wire cross section divided by available transformer window area, always <1 due to bobbin, space between turns and wire shape, isolation materials etc. typ. K_u = 0,4
W_a= transformer window area, for primary and secondary windings
A_{wp}= Primary wire cross section
A_{ws}= Secondary wire cross section

If current density is designated with J, being common for the primary and the secondary, (Amperes per unit area)

$$A_{wp} = \frac{I_p}{J}, A_{ws} = \frac{I_s}{J}$$

I_p = Primary current
I_s= Secondary current
then

$$K_u W_a = N_p \frac{I_p}{J} + N_s \frac{I_s}{J}$$

and

$$K_u W_a = \frac{V_p}{A_c \hat{B}_{ac} f K_f} \frac{I_p}{J} + \frac{V_s}{A_c \hat{B}_{ac} f K_f} \frac{I_s}{J}$$

so

$$W_a A_c = \frac{V_p I_p + V_s I_s}{K_u \hat{B}_{ac} f K_f J}$$

as

$$S_{in} = V_p I_{p,} \;\; S_o = P_o = V_s I_{s,}$$

S_{in}= Primary (input) power
S_o= Output power
I_p= Primary current (RMS)
I_s= Secondary current (RMS)
and by defining

$$S_T = S_{in} + S_0$$

$$W_a A_c = \frac{S_T}{\hat{B}_{ac}\, f\, J\, K_f\, K_u} \left[\frac{\cancel{VA}}{\frac{\cancel{Vs}}{m^2} \frac{1}{\cancel{s}} \frac{\cancel{A}}{m^2}} = m^4 \right]$$

$W_a A_c$= Area Product, a catalog value for transformer cores, defines the mechanical dimensions of the core needed on electrical (P_T,f,J,K_f) and magnetic ($\hat{B}_{ac}$,K_u) point of view.

K_f depends on the waveform of the primary voltage. Derivation of K_f for three cases (sinusoidal, square and duty cycle) has been presented in chapter 2.1.4. These cases represent typical cases of conventional (mains) transformer, push-pull converter transformer and flyback converter transformer, respectively.

S_T in the numerator depends also on the current waveform both in primary and secondary windings. Depending on the case, either one can deviate from the sinusoidal waveform increasing the value of S_t and the Area Product. This must be considered using methods presented here.

2.1.3 Calculating S_T

Case 1: Primary and secondary currents are sinusoidal. Number of turns are assumed to be equal in primary and secondary and diodes to be ideal.

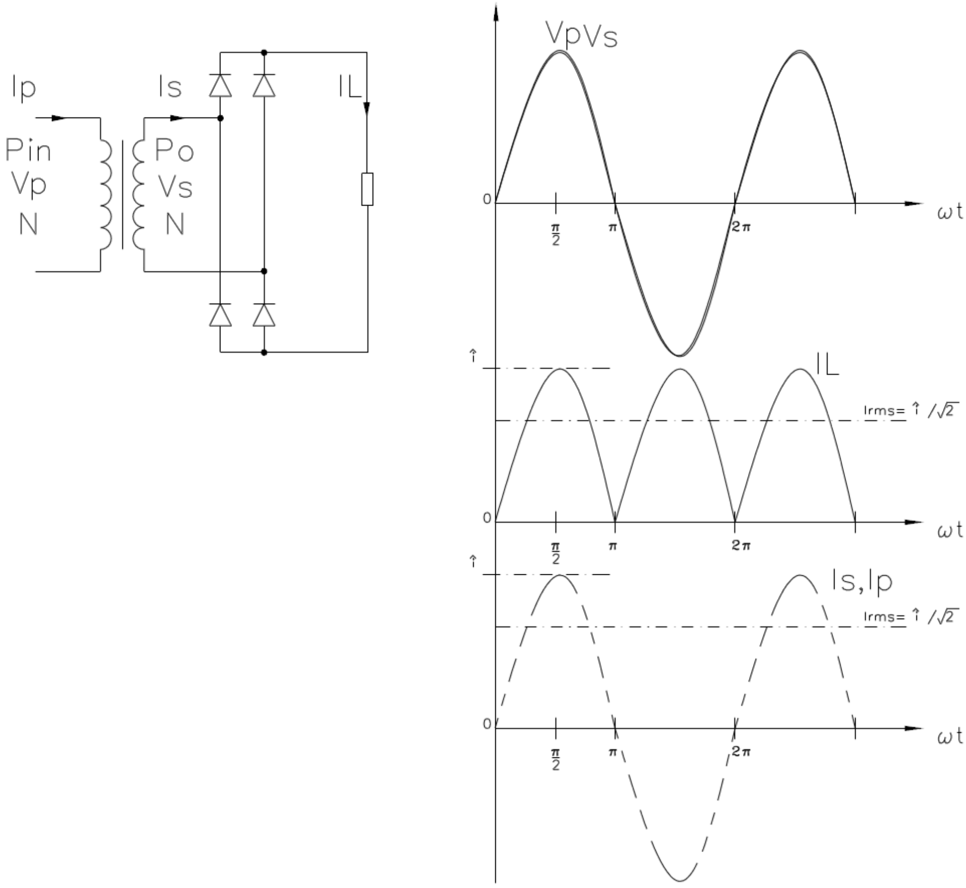


Figure 4:Transformer power rating case 1

$$S_T = S_{in} + S_0 = V_p I_p + V_s I_s$$

Case 2: Primary current is sinusoidal, secondary current in half-windings is non- sinusoidal

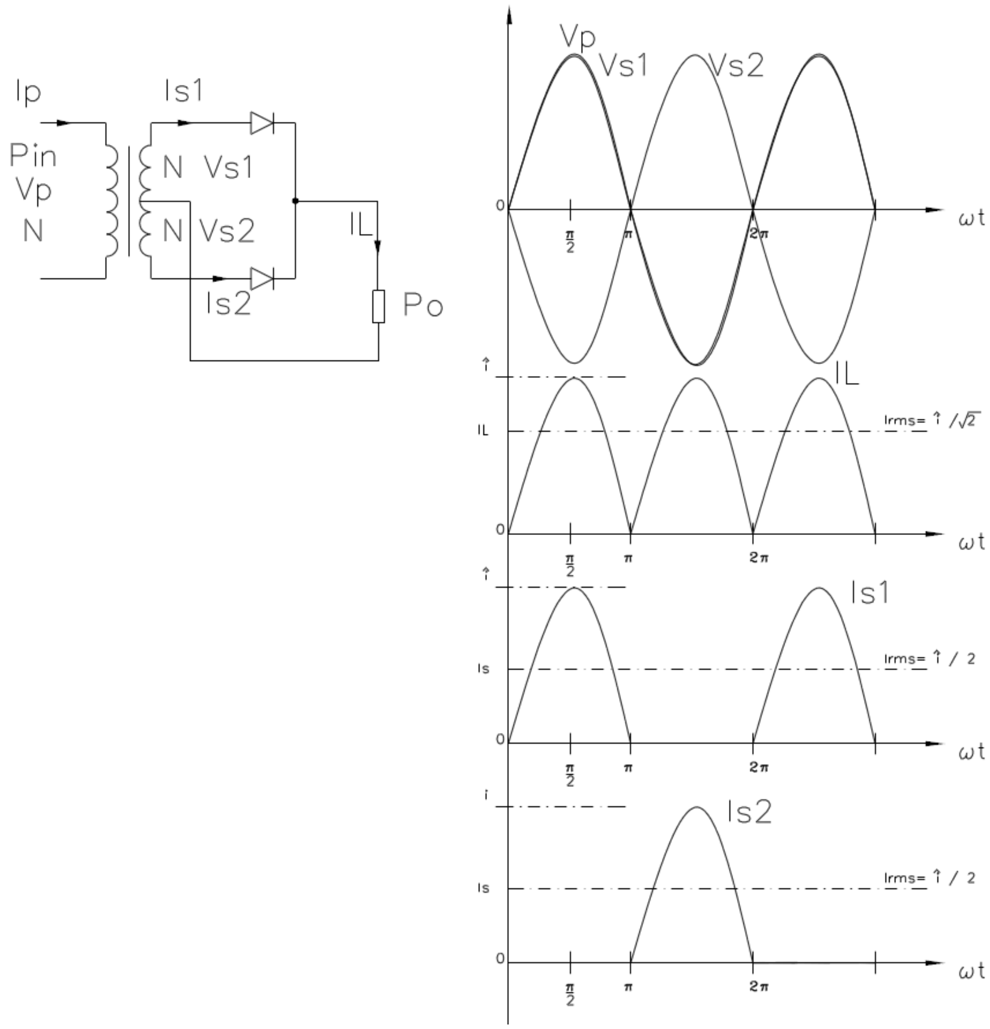


Figure 5:Transformer power rating case 2

Calculate RMS value for the non-sinusoidal secondary currents I_{s1} and I_{s2}

By definition

$$I_{s1} = I_{s2} = \sqrt{\frac{1}{2\pi}\int_0^{\pi} i^2(\omega t)d\omega t} = \sqrt{\frac{1}{2\pi}\int (\hat{\imath} sin\omega t)^2 d\omega t} = \sqrt{\frac{\hat{\imath}^2}{2\pi}\int (sin^2\omega t)d\omega t}$$

Because $\cos 2\alpha = 1 - 2sin^2\alpha \rightarrow sin^2\alpha = \frac{1-\cos 2\alpha}{2}$

$$I_{s1} = I_{s2} = \sqrt{\frac{\hat{\imath}^2}{2\pi}\int_0^{\pi}\left(\frac{1-\cos 2\omega t}{2}\right)d\omega t} = \sqrt{\frac{\hat{\imath}^2}{4\pi}\int_0^{\pi}(1-\cos 2\omega t)d\omega t}$$

Assign 2ωt=u -> $\frac{du}{2} = \frac{2d\omega t}{2} \rightarrow d\omega t = \frac{du}{2}$

$$I_{s1} = I_{s2} = \sqrt{\frac{\hat{\imath}^2}{4\pi}\int_0^{\pi}(1-\cos u)\frac{du}{2}} = \sqrt{\frac{\hat{\imath}^2}{8\pi}\int_0^{\pi}(1-\cos u)du} = \sqrt{\frac{\hat{\imath}^2}{8\pi}\left[\int_0^{\pi}1du - \int_0^{\pi}cosu\ du\right]}$$

$$= \sqrt{\frac{\hat{\imath}^2}{8\pi}\left[\overset{\pi}{\underset{0}{|}}u - \overset{\pi}{\underset{0}{|}}\sin u\right]} = \sqrt{\frac{\hat{\imath}^2}{8\pi}\left[\overset{\pi}{\underset{0}{|}}2\omega t - \overset{\pi}{\underset{0}{|}}\sin 2\omega t\right]} = \sqrt{\frac{\hat{\imath}^2}{8\pi}[(2\pi - 0) - (sin2\pi - sin0)]} = \frac{\hat{\imath}}{2}$$

Therefore,

$$S_T = S_{in} + S_0 = V_pI_p + V_sI_s = V_pI_p + (V_sI_{s1} + V_sI_{s2}) = V_pI_p + V_s\frac{\hat{\imath}}{2} + V_s\frac{\hat{\imath}}{2}$$
$$= V_pI_p + \frac{V_s\sqrt{2}I_s}{2} + \frac{V_s\sqrt{2}I_s}{2} = V_pI_p + \frac{2V_s\sqrt{2}I_s}{2} = V_pI_p + V_s\sqrt{2}I_s$$

Because $V_p=V_s$ and $I_p= I_s$

$$S_T = V_pI_p(1+\sqrt{2})$$

S_t becomes 20,7% larger than in case 1.

2.1.4 Derivation of waveform factors K_f

Case 1: Voltage sinusoidal, -> flux sinusoidal

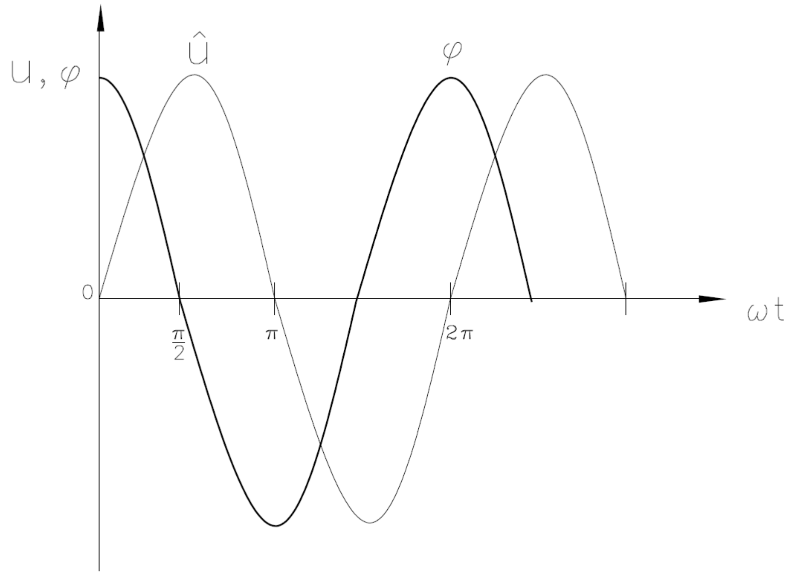


Figure 6:Voltage sinusoidal, flux sinusoidal

$$U = -N\frac{d\varphi}{dt}$$

When $u(t) = \hat{u}\sin(\omega t)$, $\hat{u}\sin(\omega t) = -N\frac{d\varphi}{dt} \rightarrow \frac{d\varphi}{dt} = \frac{\hat{u}\sin(\omega t)}{-N}$

During a half-cycle

$$\phi_{ave} = \frac{2\hat{u}}{-N\pi}\int_0^{\pi/2}\sin(\omega t)dt = \frac{2\hat{u}}{-N\pi\omega}\Big|_0^{\pi/2} - \cos(\omega t) = \frac{2\hat{u}}{N\pi\omega}$$

On the other hand,

$$\phi_{ave} = \frac{2}{\pi}\int_0^{\pi/2}\hat{\varphi}\sin(\omega t)dt = \frac{2\hat{\varphi}}{\pi}\Big|_0^{\pi/2} - \cos(\omega t) = \frac{2\hat{\varphi}}{\pi} = \frac{2\sqrt{2}\phi}{\pi}$$

ϕ= RMS- value of the flux,

so

$$\phi = \frac{\cancel{2}\hat{u}}{N\cancel{\pi}\omega}\frac{\cancel{\pi}}{\cancel{2}\sqrt{2}} = \frac{\hat{u}}{\sqrt{2}N\omega} = \frac{U}{N\omega};\ \omega = 2\pi f,$$

$$U = \omega N\,\phi = \frac{2\pi}{\sqrt{2}}fN\hat{\varphi} = K_f f N\hat{\varphi} \approx 4{,}44\ f\ N\ \hat{\varphi}$$

U = RMS- value of the voltage.

Case 2: Voltage square, -> flux triangular (Push-pull- switching)

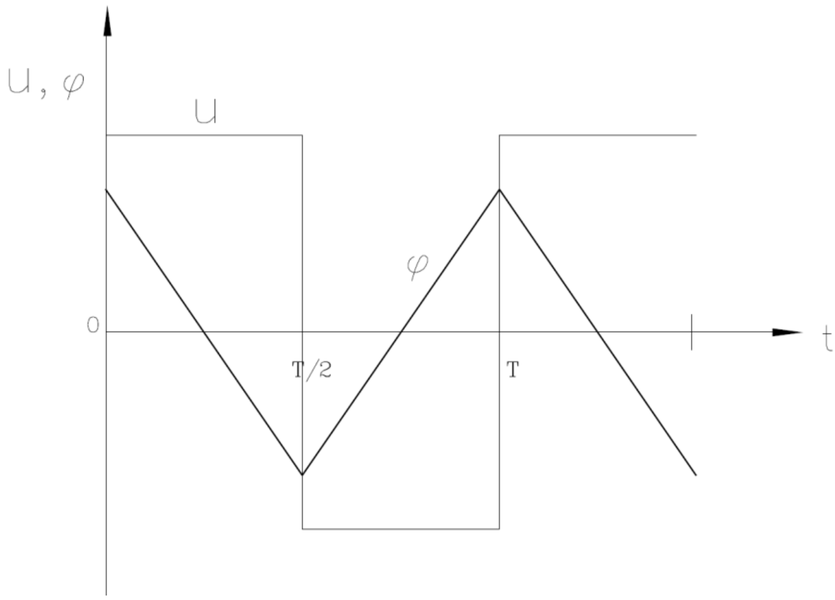


Figure 7:Voltage square, flux triangular

$$U = -N\frac{\Delta\varphi}{\Delta t} = N\frac{2\hat{\varphi}}{\frac{T}{2}} = N\frac{4\hat{\varphi}}{T} = K_f f N\hat{\varphi} = 4fN\hat{\varphi}$$

Case 3: Voltage duty-cycled, -> flux sawtooth (Flyback converter)

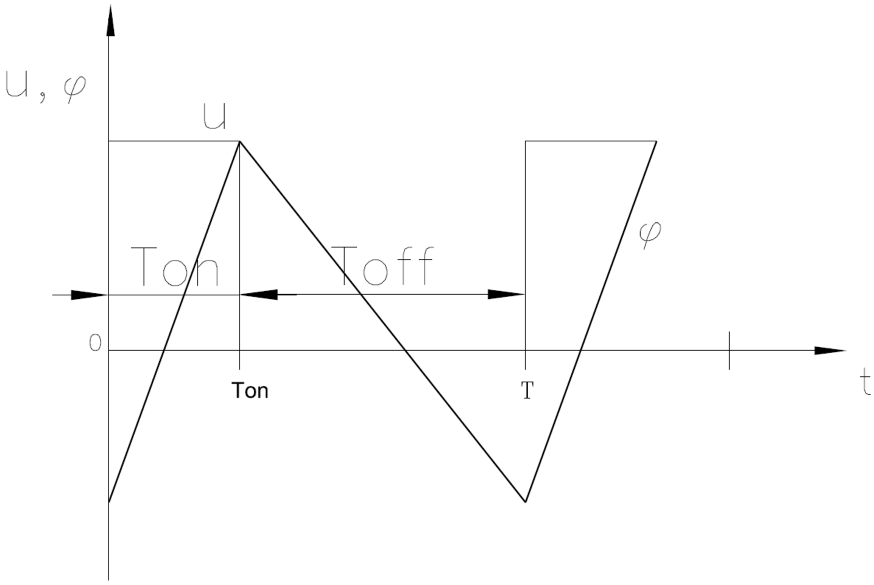
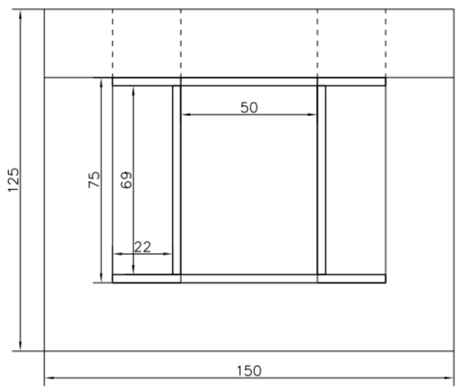


Figure 8:Voltage duty-cycled, flux sawtooth

$$D = \frac{T_{on}}{T_{on} + T_{off}} = \frac{T_{on}}{T}$$

$$U = -N\frac{\Delta\varphi}{\Delta t} = N\frac{2\hat{\varphi}}{T_{on}} = N\frac{2\hat{\varphi}}{DT} = K_f f N\hat{\varphi} = 2\frac{T}{T_{on}} f N\hat{\varphi}$$

A transformer with following laminated iron core dimensions (in mm) and name plate information was available in this example case:



Prim: 230V
Sek: 48V 16A
750 VA
50-60 Hz

$$A_p = A_c A_w = \frac{S_T}{\hat{B}_{ac}\, f\, J\, K_f\, K_u} \quad [m^4]$$

where

A_p= Area product (m^4)
A_c= Core area (m^2)
A_w= Winding window area (m^2)
S_T =S_{in} + S_o (W)
K_f= Waveform coefficient, 4.44 for sine wave, 4.0 for square wave
K_u = Utilization factor
$\hat{B}_{ac}$= Flux density peak value (T=Vs/m^2)
J= Current density (A/m^2)
f= Frequency (1/s =Hz)

2.1.5 Calculate necessary A_p

As transformer primary and secondary currents are sinusoidal, the requirement is:

$$A_p \geq \frac{\frac{450}{0{,}9} + 450\ \cancel{VA}}{4{,}44 * 0{,}4 * 1{,}1 \frac{\cancel{Vs}}{m^2} * 2{,}5 * 10^6 \frac{\cancel{A}}{m^2} * 50 \frac{1}{\cancel{s}}} = 3{,}89 * 10^{-6} m^4$$

For the available core:

$$A_p = A_c A_w = (80 * 10^{-3} * 50 * 10^{-3})(69 * 10^{-3} * 22 * 10^{-3}) m^4 = 6{,}07 * 10^{-6} m^4$$

So, the core size is sufficient.

2.1.6 Calculate number of turns in the primary winding

$$N_p = \frac{V_p}{A_c \hat{B}_{ac} f K_f} = \frac{230\cancel{V}}{80 * 10^{-3} * 50 * 10^{-3}\cancel{m^2} * 1{,}1\frac{\cancel{Vs}}{\cancel{m^2}} * 50\frac{1}{\cancel{s}} * 4{,}44} \approx 235$$

2.1.7 Calculate number of turns in the secondary winding:

$$N_s = \frac{U_s}{U_p} N_p = \frac{18V}{230V} \approx 18$$

2.1.8 Calculate wire cross section

Primary:

$$A_{wp} \geq \frac{\frac{\frac{450}{0{,}9}VA}{230V}}{\frac{2{,}5A}{mm^2}} = 1{,}1\,mm^2 \rightarrow 1{,}5mm^2$$

Secondary:

$$A_{ws} \geq \frac{30A}{\frac{2{,}5A}{mm^2}} = 12\,mm^2$$

Secondary winding of the existing transformer was removed and replaced with a new one consisting of two parallel-connected windings 18 turns each. Cross section of enameled wire in both secondary windings was $6mm^2$. Primary side needed no change and was left as it was before.

3. Schematic

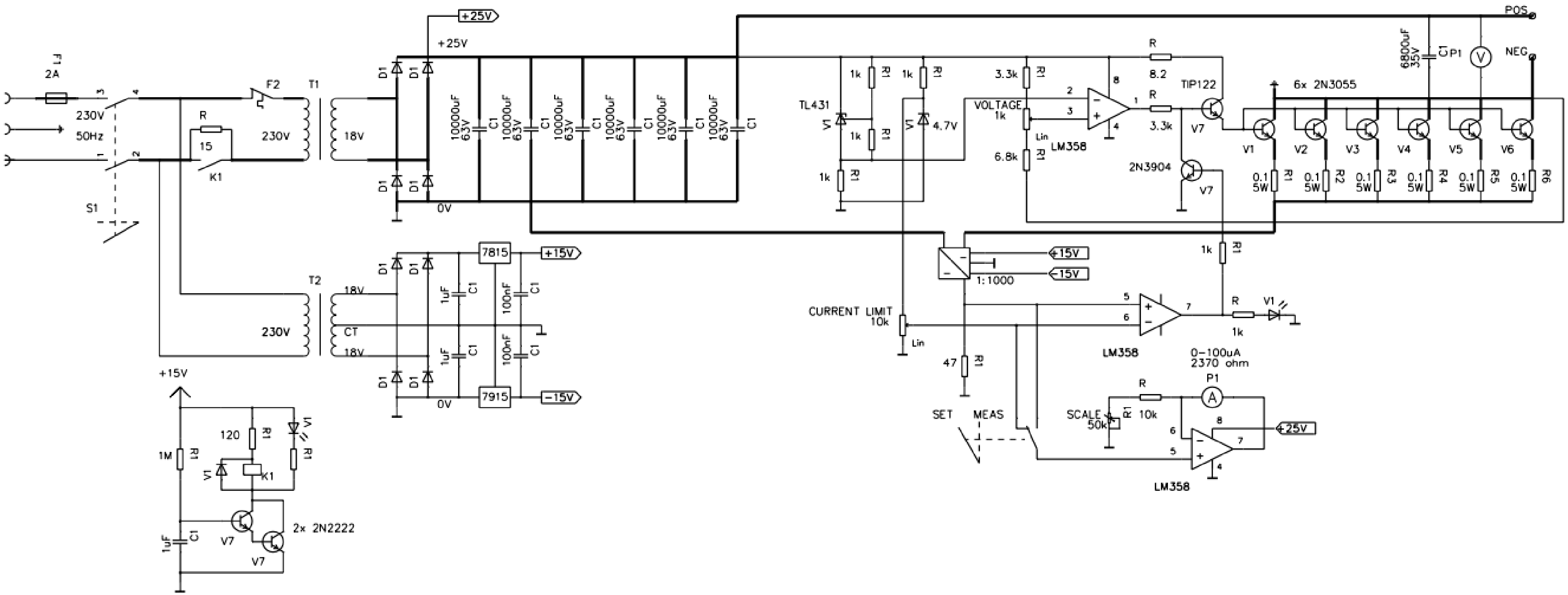


Figure 9:Linear power supply schematic

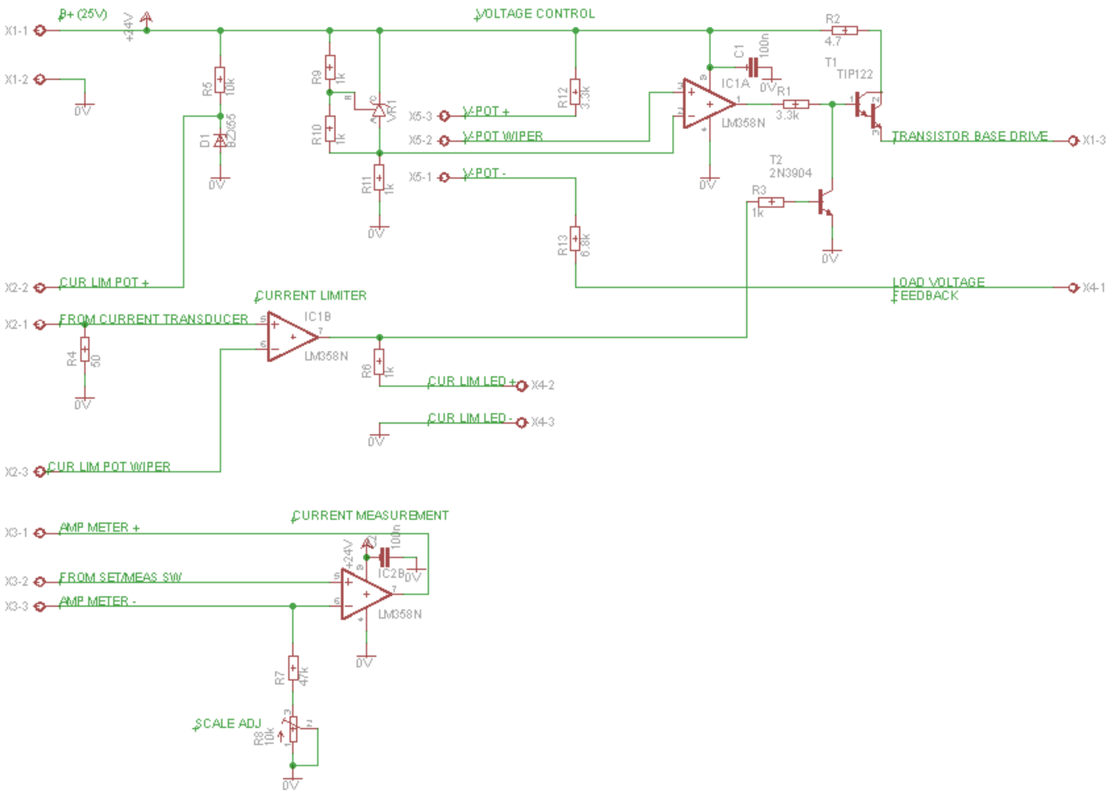


Figure 10:Control circuit

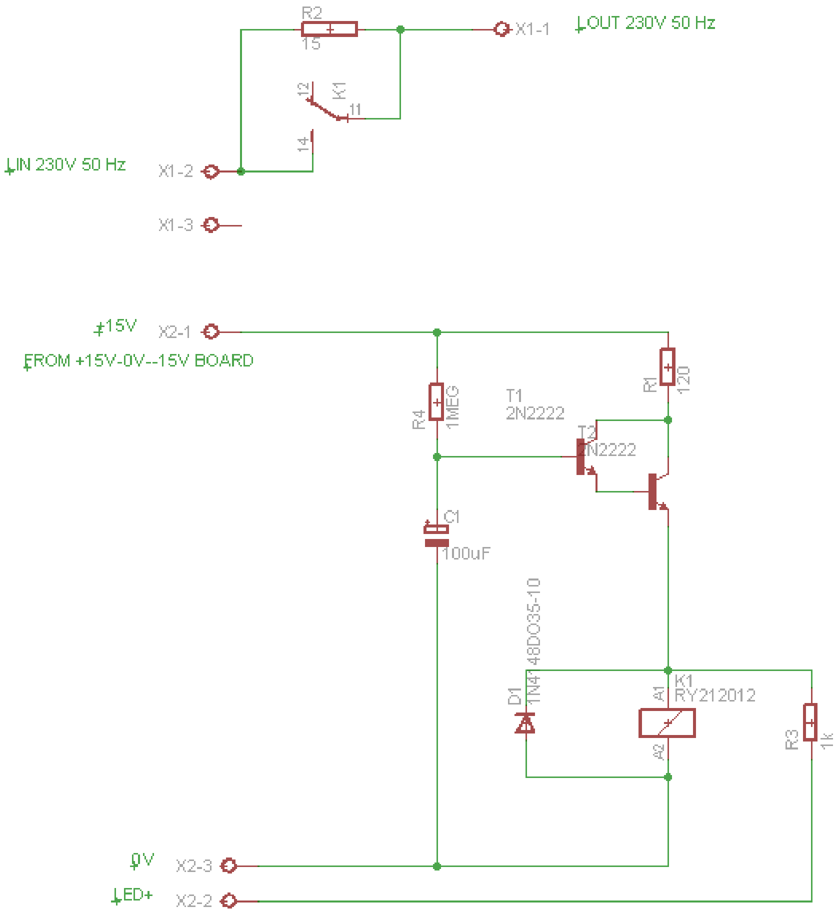
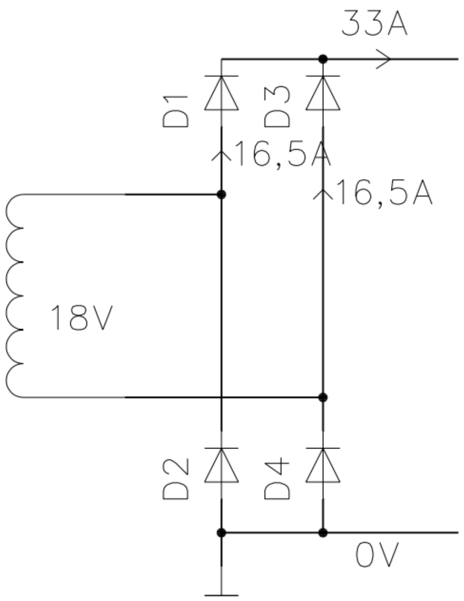


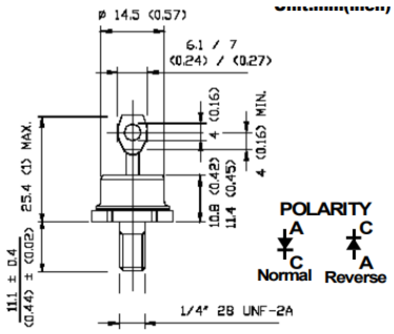
Figure 11: Soft start circuit

3.1 Rectifiers



Current to load and regulator transistor base drive is approximately 33A at full load. Diode forward average current $I_{F(AV)}$ will be half of that, 16,5A.

A suitable diode 70HF120 has $I_{F(AV)}$ 180° conduction, half sine wave =70 A. Maximum peak, one cycle forward,10ms, surge current I_{FSM}=1000A (100% V_{RRM} reapplied). Maximum repetitive reverse voltage V_{RRM}=1200V.



3.2 Soft start

To limit diode surge current a resistor shall be placed in the transformer primary for initial current limiting.

Resistance in the secondary side should be more than

$$R_s \geq \frac{Us}{I_{FSM}} = \frac{25V}{1000A} \approx 25m\Omega$$

Resistance reduced to transformer primary side:

$$R_p = R_s\mu^2 = R_s\left(\frac{U_p}{U_s}\right)^2 = 21 * 10^{-3}\frac{V}{A} * \left(\frac{230}{18}\right)^2 \approx 4{,}1\Omega$$

3.3 Filter capacitors

Select allowable ripple ΔU in filtered voltage (this is not load voltage ripple) at full load I_{dc}= 30 A@21 V => 3,5$V_{p\text{-}p}$.

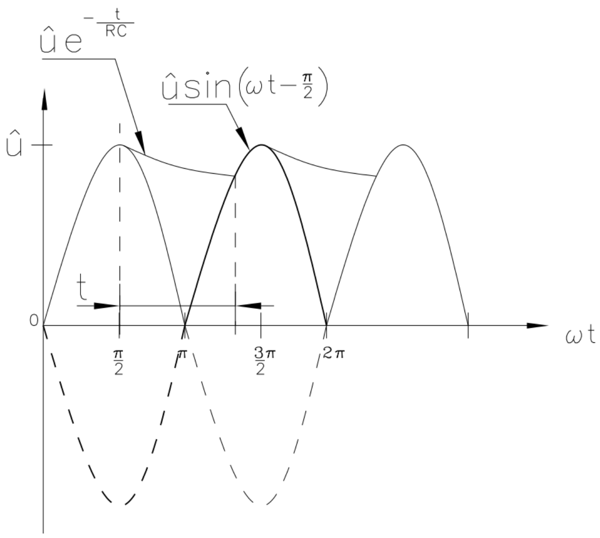


Figure 12:Supply voltage filtering

First, discharge time t can be assumed to be a half-cycle, 10ms at 50 Hz.

As capacitor charge Q and capacitance C can be expressed as

$$Q = I_{dc}t$$

$$C = \frac{Q}{\Delta U} = \frac{I_{dc}t}{\Delta U} = \frac{30A * 10 * 10^{-3}s}{3{,}6V} \approx 83\,000\,\mu F$$

To calculate filter capacitor discharge time t more precisely, the following equation can be written at the end of the discharge cycle

$$\hat{u}e^{-\frac{t}{RC}} = \hat{u}\sin(\omega t - \frac{\pi}{2})$$

Where

û= rectifier supply voltage peak value
R = load resistance
C= filter capacitor capacitance

To solve t , e^x and sin(x) can be expressed as

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \cdots$$

and

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} + \frac{x^7}{7!} + \cdots$$

If x is small,

$$e^x \approx 1 + x$$
$$\sin x \approx x$$

Therefore

$$\cancel{\hat{u}}e^{-\frac{t}{RC}} = \cancel{\hat{u}}\sin(\omega t - \frac{\pi}{2}) \rightarrow$$

$$1 - \frac{t}{RC} = \omega t - \frac{\pi}{2} =>$$

$$t = \frac{1 + \frac{\pi}{2}}{\omega + \frac{1}{RC}} = \frac{1 + \frac{\pi}{2}}{2\pi 50\frac{1}{s} + \frac{1}{0{,}7\frac{V}{A} * 83300 * 10^{-6}\frac{As}{V}}} \approx 7{,}7ms$$

Using this discharge time t, C becomes

$$C = \frac{I_{dc}t}{\Delta U} = \frac{30A * 7{,}7 * 10^{-3}s}{3{,}6V} \approx 64000\,\mu F$$

To round off to an available component size, 6 x 10 000 µF capacitors were selected.

3.4 Output regulation

3.5 Current sense

3.6 Current measurement and current limit

3.7 Short circuit protection

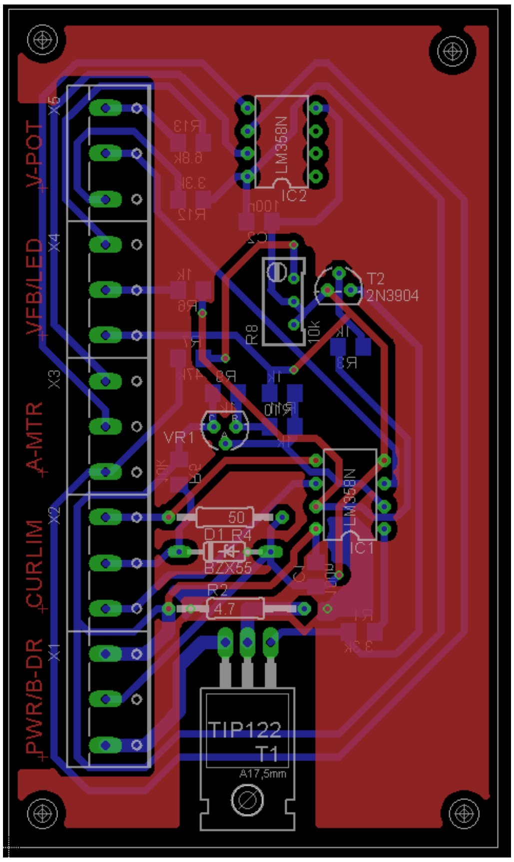
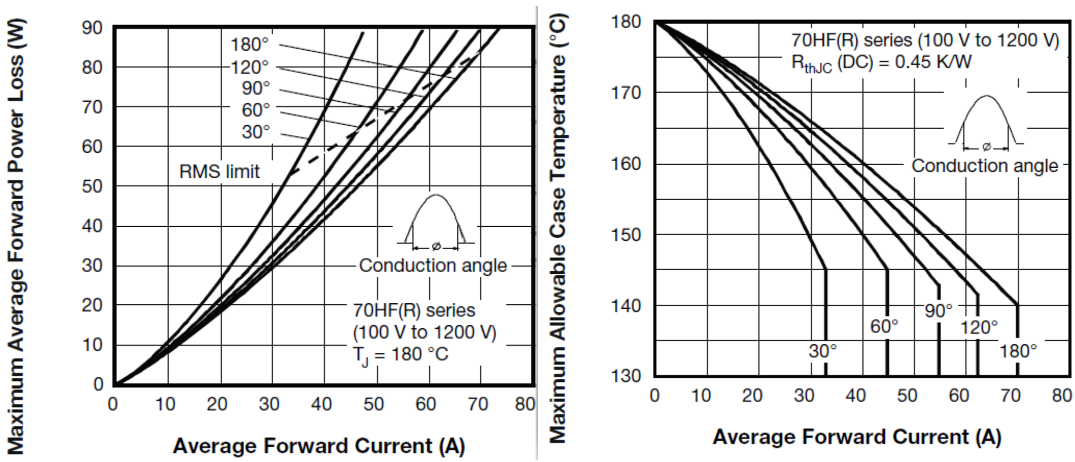


Figure 13: Control board PCB

4. Thermal design

4.1 Rectifier bridge

Forward power loss P_h at 16,5A is about 15W/diode as can be observed in the datasheet outtake below.



Thermal resistance from junction to case of the diode for DC is $R_{thJC(DC)}$=0,45 K/W. For sinusoidal 180° conduction, R_{thJC} increases by 0,08 K/W-> R_{thJC}=0,53 K/W. Thermal resistance from case to heatsink when mounting surface is smooth, flat and greased, R_{thCH} = 0,25 K/W. Maximum case temperature is about 172 °C at sinusoidal 180° conduction. Design value for case temperature is 120 °C and 35°C for the ambient temperature.

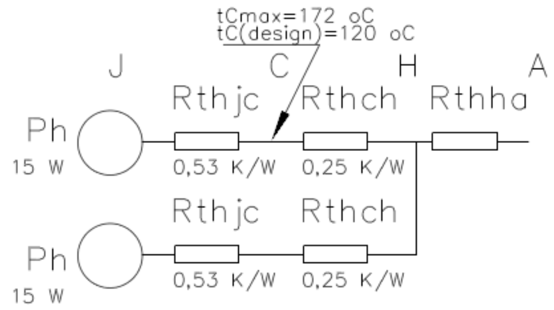
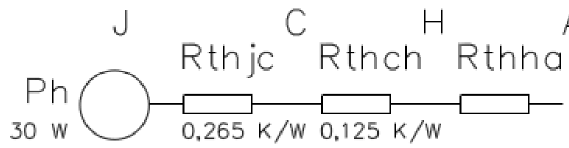


Figure 14:Thermal equivalent circuit for two diodes on a common heatsink

Total power loss and resulting thermal resistances for two diodes (R_{thjc}'s and R_{thch}'s are in parallel)



Junction temperature and temperature difference ΔT from junction to ambient can be calculated.

$$\Delta T = \left(T_C + 2P_h\frac{R_{thjc}}{2}\right) - T_A = (120+273)K + 2 * 15W\,\frac{0{,}53}{2}\frac{K}{W} - (273+35)K \approx 93K$$

and requirement for the heatsink:

$$R_{thha} \leq \frac{\Delta T - P_h\left(R_{thjc} + R_{thch}\right)}{P_h} = \frac{93K - 30W\,(0{,}265+0{,}125)\,\frac{K}{W}}{30W} = 2{,}7\;K/W$$

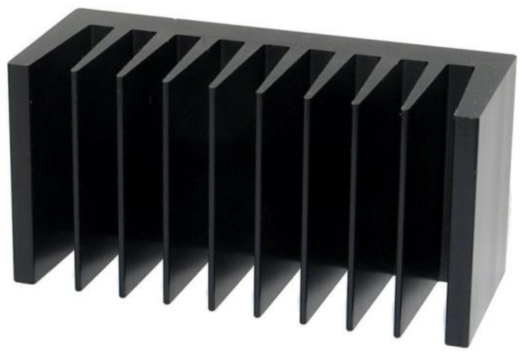


Figure 15: Heatsink 1,9 K/W (Fischer SK 92/50 SA)

2 pcs 100x50x40mm heatsinks with thermal resistance R_{thha} = 1,9 K/W were selected, one for each type (bolt to anode/bolt to cathode) of diode pairs. Both positive and negative heatsinks need to be

electrically isolated from the power supply metal chassis, which is the negative terminal of the load voltage.

4.2 Output stage

As the load voltage regulation is based on the voltage drop across collector-emitter of the output transistors, and when only a single output voltage from the transformer secondary is available, allowable load current at a given load voltage will be limited by the amount of heat that can be dissipated by the heat sinks (at lower load voltages) and also by the current capacity of the secondary winding of the transformer (at higher load voltages).

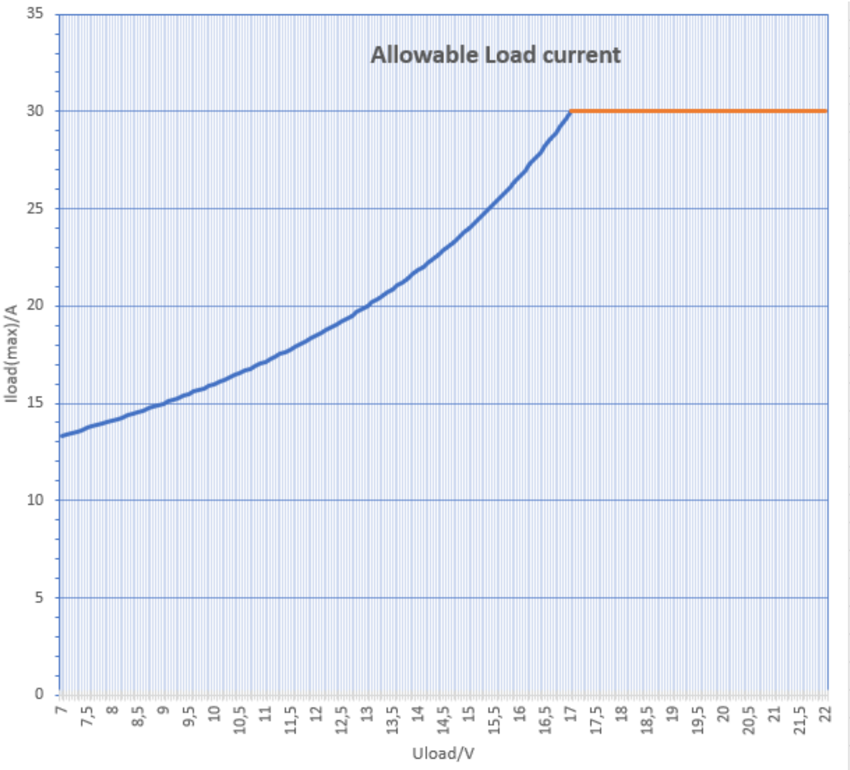


Figure 16:Allowable load current (calculated with selected heat sink 2,2 K/W)

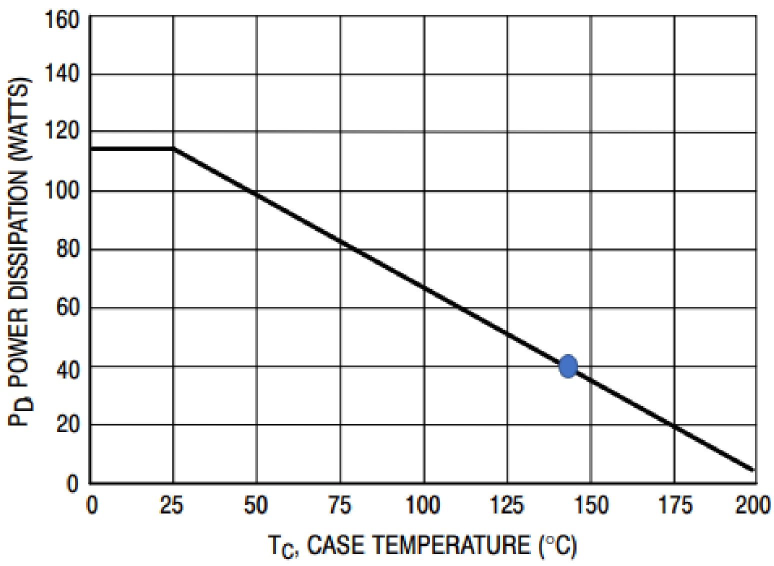
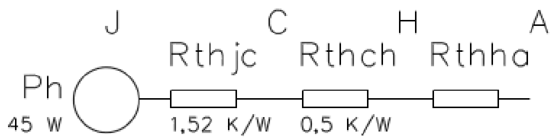


Figure 17:2N3055 Power dissipation derating

To start calculation for a proper heat sink, a point on the derating curve of 2N3055 transistor is selected. When transistor case temperature T_C is 130 °C, allowable power dissipation ($P_d=I_c * U_{CE}$) is about 40W. R_{thJC}=1,52 K/W. R_{thch} is assumed to be 0,5 K/W (with thermal paste, no insulation). Ambient temperature is assumed to be 25°C.



Junction temperature and temperature difference ΔT from junction to ambient can be calculated.

$$\Delta T = \left(T_C + P_h R_{thjc}\right) - T_A = (130 + 273)K + 38W * 1{,}52\frac{K}{W} - (273 + 25)K \approx 163\ K$$

Requirement for the heat sink:

$$R_{thha} \leq \frac{\Delta T - P_h\left(R_{thjc} + R_{thch}\right)}{P_h} = \frac{168K - 38W(1{,}52 + 0{,}5)\frac{K}{W}}{38W} = 2{,}26\ K/W$$

Figure 18:Heatsink 2,2 K/W (Fischer SK 08/50 SACB)

6pcs 100x50x40mm heatsinks with thermal resistance R_{thha} = 2,2 K/W were selected.

Safe operating area of the transistor is confirmed next. To avoid so-called secondary breakdown, collector current and collector-emitter voltage shall be situated within transistor's safe operating area. As the collector current per transistor does not exceed 5A at any design point, collector-emitter voltage may be ≤25V at DC, so this parameter is OK.

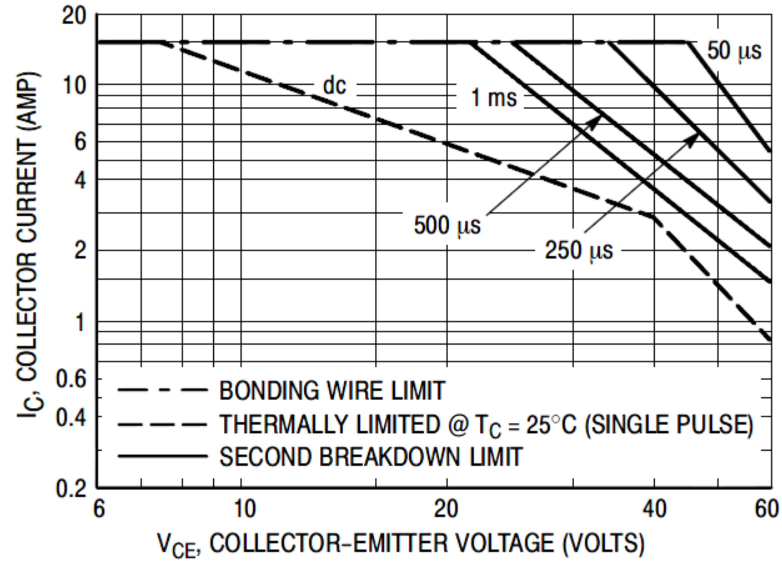


Figure 19:2N3055 Safe operating area

Vaihtojännitteen tehollisarvo on arvoltaan sama kuin sellainen tasajännite, joka resistiivisessä kuormassa (vastuksessa) tuottaa yhtä suuren tehon. Koska vastuksessa kuluva teho on verrannollinen jännitteen neliöön $P=U^2/R$, vaihtojännitteen hetkellinen teho on

$$P(t) = \frac{u^2(t)}{R}$$

Verkkojännite on sinimuotoinen, û= jännitteen huippuarvo, ωt=2πf kulmataajuus ja f on taajuus (Hz)

$$u = \hat{u}\, sin\omega t$$

Ajan T kuluessa vastuksessa R vaikuttaa keskimäärin teho Pavg, joka tehollisarvon määritelmän mukaan on sama kuin tasajännitteen U samassa vastuksessa aiheuttama teho

$$P_{avg} = \frac{1}{R}\left(\frac{1}{T}\int_0^T u^2(t)dt\right) = \frac{U^2}{R}$$

ratkaisemalla U saadaan

$$U = \sqrt{\frac{1}{T}\int_0^T u^2(t)dt}$$

josta havaitaan, jännitteen tehollisarvo on vaihtojännitteen neliön keskiarvon neliöjuuri, jota kutsutaan myös RMS (root-mean-square) –arvoksi.

Koska vaihtojännitteen puolijaksot ovat symmetriset, riittää, kun tarkastellaan yhtä puolijaksoa (väliä ωt=0…π).

$$U = \sqrt{\frac{1}{T}\int_0^T u^2(t)dt)} = \sqrt{\frac{1}{\pi}\int (\hat{u}sin\omega t)^2 d\omega t} = \sqrt{\frac{\hat{u}^2}{\pi}\int (sin^2\omega t)d\omega t}$$

Koska $\cos 2\alpha = 1 - 2sin^2\alpha \rightarrow sin^2\alpha = \frac{1-\cos 2\alpha}{2}$

$$U = \sqrt{\frac{\hat{u}^2}{\pi}\int_0^\pi \left(\frac{1-\cos 2\omega t}{2}\right) d\omega t} = \sqrt{\frac{\hat{u}^2}{2\pi}\int_0^\pi (1-\cos 2\omega t)d\omega t}$$

Merkitään 2ωt=u -> $\frac{du}{2} = \frac{2d\omega t}{2} \rightarrow d\omega t = \frac{du}{2}$

$$U = \sqrt{\frac{\hat{u}^2}{2\pi}\int_0^\pi (1-\cos u)\frac{du}{2}} = \sqrt{\frac{\hat{u}^2}{4\pi}\int_0^\pi (1-\cos u)du} = \sqrt{\frac{\hat{u}^2}{4\pi}\left[\int_0^\pi 1du - \int_0^\pi cosu\ du\right]}$$

$$= \sqrt{\frac{\hat{u}^2}{4\pi}\left[\Big|_0^\pi u - \Big|_0^\pi \sin u\right]} = \sqrt{\frac{\hat{u}^2}{4\pi}\left[\Big|_0^\pi 2\omega t - \Big|_0^\pi \sin 2\omega t\right]} = \sqrt{\frac{\hat{u}^2}{4\pi}[(2\pi - 0) - (sin2\pi - sin0)]} = \frac{\hat{u}}{\sqrt{2}}$$